Upper tails for counts of random objects

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Abstract

Let $\mathcal{S}$ be a family of subsets of a finite set Γ and let Γ_p be a binomial random subset of Γ. Further, let X count the number of members of $\mathcal{S}$ contained in Γ_p. A lot of research has been devoted to the study of the asymptotic distribution of X when $N = |\Gamma| \to \infty$ and $p = p(N)$, both, in the general setting and for special instances, most notably for random graphs.

One feature which has received a lot of attention is the rate of decay of the tails of X, the lower tail $Prob(X \leq tEX)$ for $0 < t < 1$, and the upper tail $Prob(X \geq tEX)$ for $t > 1$. Good estimates for the lower tail follow from the FKG inequality (lower bound) and Janson's inequality (upper bound).

The upper tails tend to be harder to analyze. For the subgraph count problem a quite satisfactory and complete result was obtained by Janson, Oleszkiewicz, and Ruciński, where the logarithms of the upper and lower bound on $Prob(X \geq tEX)$ are of the same order of magnitude except for a logarithmic term. A generalization to random hypergraphs was carried over by Dudek, Polcyn, and Ruciński.

In this talk I will show how, using the proof techniques developed earlier, those results are extended in two directions. First, we examine the more general model of set systems (hypergraphs) and obtain some straightforward estimates for the upper tail of X, covering, among others, the number of arithmetic progressions of given length in a random subset of integers. Then, we return to the subgraph counts in random graphs to study the *rooted* version of the problem, only to discover some unexpected features there. This is joint work with Svante Janson.