

ACO Seminar

Nov. 4, 3:30pm, Wean 8220

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Diamond-free Families

Abstract:

Given a finite poset P , we consider the largest size $\text{La}(n, P)$ of a family of subsets of $[n] := \{1, \dots, n\}$ that contains no subposet P . Sperner's Theorem (1928) gives that $\text{La}(n, P_2) = \binom{n}{\lfloor n/2 \rfloor}$, where P_2 is the two-element chain. This problem has been studied intensively in recent years, and it is conjectured that $\pi(P) := \lim_{n \rightarrow \infty} \text{La}(n, P) / \binom{n}{\lfloor n/2 \rfloor}$ exists for general posets P , and, moreover, it is an integer. For $k \geq 2$ let D_k denote the k -diamond poset $\{A < B_1, \dots, B_k < C\}$. We study the average number of times a random full chain meets a P -free family, called the Lubell function, and use it for $P = D_k$ to determine $\pi(D_k)$ for infinitely many values k . A stubborn open problem is to show that $\pi(D_2) = 2$; here we prove $\pi(D_2) < 2.273$ (if it exists).

Joint work with Wei-Tian Li, and Linyuan Lu